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Spin Dependence of D0-brane Interactions

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The long-range, spin-dependent forces between D0-branes are related to long-range fundamental string interactions using duality. These interactions can then be computed by taking the long distance non-relativistic expansion of string four-point amplitudes. The results are in accord with the general constraints of Matrix Theory.

1. Introduction

It is commonly stated that there is a vanishing static force between BPS states in supersymmetric theories and this is reflected in the fact that there often exist exact multiparticle solutions to the classical BPS equations for supersymmetric configurations. This simple picture is however modified by quantum and relativistic effects. Quantum effects associated with fermion zero modes give soliton multiplets with spin which lie in representations of the spacetime supersymmetry algebra. Spin dependent static forces between BPS states are then formally of the same order in an expansion in powers of $1/c$ as velocity dependent interaction terms which are independent of spin, a fact which should be familiar from the study of relativistic corrections to the hydrogen atom spectrum where p^4 corrections to the Hamiltonian and spin-orbit interactions arise at the same order in $1/c$. Thus we must include these spin-dependent interactions to have a consistent supersymmetric description of the long-range interactions between BPS states.

An example of this phenomenon occurs in the long-range interactions between magnetic monopoles in $N = 4$ gauge theory. The supersymmetric quantum mechanics on the two monopole moduli space contains a four fermion term proportional to the curvature tensor on the moduli space. Upon quantizing the fermion fields this interaction gives a static spin-dependent interaction between two monopoles. The existence of this long-range interaction is crucial for the existence of BPS bound states in this system as can be seen from the analysis in [1].

In this talk I will discuss the long-range spin dependent interactions between D0-branes in type IIA string theory.

2. D0-branes and duality

We first recall a few basic facts about D0-branes. These are BPS states which fall into a 256 dimensional short representation of the $N = (1, 1)$ supersymmetry algebra in $D = 10$. This representation consists of states transforming as $128 \oplus 84 \oplus 44$ under the $SO(9)$ rotation group. The D0-branes can be viewed as the Kaluza-Klein momentum modes of the Rarita-Schwinger field (ψ_M), third rank antisymmetric tensor field (A_{MNP}), and graviton field (g_{MN}) of $D = 11$ supergravity compactified on S^1 [2,3].

The leading spin independent static force between D0-branes vanishes as shown in [4]. The leading non-zero long-range interaction between D0-branes goes as v^4/r^7 with v the D0-brane velocity and r their separation [5]. The fact that Matrix theory reproduces this interaction through a one-loop calculation in quantum mechanics was one of the first pieces of evidence for the correctness of the proposal of [6]. The v^4/r^7 interaction is universal, that is independent of the

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spin state of the D0-brane. There are a number of approaches to computing the spin dependent long-range interactions. At large distances only exchanges of massless states contribute so one needs only to compute the t-channel diagrams from exchange of the massless fields of IIA supergravity. The various normalizations that enter into such a computation are most easily organized by performing the computation directly in string theory. This could be done by generalizing the computation of [4] to include the effects of fermion boundary states. Here we will take a more indirect route using string duality. The spin-dependent interactions should also follow from Matrix theory by generalizing the computation of [5] to include the terms which are required to supersymmetrize the v^4/r^7 interaction.

To compute the spin dependent interactions we first compactify the IIA theory on a S^1 of radius R_9 with coordinate x_9. A T-duality transformation along x_9 turns the D0-brane into a D1-brane wrapped on the S^1 in IIB theory. An S-duality transformation then turns the D1-brane into a fundamental IIB string wrapped on the S^1. Since the long-range forces between BPS states are dictated by their coupling to massless states and these couplings are protected by supersymmetry, the long-range interaction between D0-branes can be computed by computing the long-range interactions between wound IIB fundamental strings. One disadvantage of this approach is that massive states in $D = 9$ are classified by $SO(8)$ so that the answer will be expressed in terms of $SO(8)$ invariants rather than $SO(9)$ invariants.

3. Kinematics

For IIB string on S^1 half-saturated BPS states satisfy $N_L = N_R = 0$ (in Green-Schwarz formalism) and hence $mn = 0$ with m the momentum and n the winding on S^1. We consider $m = 0$ and n arbitrary. Under the duality described above n labels the D0-brane charge. The $SO(9)$ spin states for D0-branes decompose under $SO(9) \to SO(8)$ as

$$\mathbf{44} \quad \to \quad (\mathbf{1} + \mathbf{35_v})_{NSNS} + (\mathbf{8_v})_{RR} \tag{1}$$

$$\mathbf{84} \quad \to \quad (\mathbf{28})_{NSNS} + (\mathbf{56_v})_{RR} \tag{2}$$

$$\mathbf{128} \quad \to \quad (\mathbf{8_c} + \mathbf{8_s} + \mathbf{56_c} + \mathbf{56_s})_{NSR+RNS} \tag{3}$$

where the subscripts indicate which sector (RR, NSNS, RNS, NSR) of IIB string theory the states come from.

We want to compute four-point string amplitudes for the scattering of such states. These states are labelled by the left and right-moving spinors or polarization vectors to be discussed momentarily and by their ten-dimensional momenta

$$p_1^{R,L} \quad = \quad (\frac{1}{2}\hat{p}_1, \pm nR) \tag{4}$$

$$p_2^{R,L} \quad = \quad (\frac{1}{2}\hat{p}_2, \pm nR) \tag{5}$$

$$p_3^{R,L} \quad = \quad (\frac{1}{2}\hat{p}_3, \mp nR) \tag{6}$$

$$p_4^{R,L} \quad = \quad (\frac{1}{2}\hat{p}_4, \mp nR) \tag{7}$$

Here the hatted quantities are nine-dimensional momenta which in the center of mass frame are given by

$$\hat{p}_1 \quad = \quad E(1, \vec{v}), \qquad \hat{p}_2 = E(1, -\vec{v}) \tag{8}$$

$$\hat{p}_3 \quad = \quad -E(1, \vec{w}), \qquad \hat{p}_4 = -E(1, -\vec{w}) \tag{9}$$

with $|\vec{v}| = |\vec{w}| = v$, $\vec{v} \cdot \vec{w} = v^2 \cos\theta$, and $E^2(1 - v^2) = M^2 \equiv 4n^2R^2$. It will also be useful to define the eight-momentum transfer $\vec{q} = M(\vec{v}+\vec{w})$ and the vector $\vec{k} = M(\vec{v} - \vec{w})$ which as $\vec{q}$ goes to zero reduces to $2M\vec{v}$ with $\pm\vec{v}$ the velocity of the incoming D0-branes.

Note that the ten-dimensional Mandelstam invariants are the same for left and right-movers since there are no directions with both momentum and winding.

We also define nine-dimensional Mandelstam invariants

$$\hat{s} \quad \equiv \quad -(\hat{p}_1 + \hat{p}_2)^2 \tag{10}$$

$$\hat{t} \quad \equiv \quad -(\hat{p}_2 + \hat{p}_3)^2 \tag{11}$$

$$\hat{u} \quad \equiv \quad -(\hat{p}_1 + \hat{p}_3)^2 \tag{12}$$

3.1. Polarization vectors and spinors

We will also need some details about polarization vectors and spinors. With the above kinematics we can consider a t-channel process in

which particle 1 exchanges massless fields with particle 2 and the particles labelled 4 and 3 are the final states of particles 1 and 2 respectively. To compute the long-range force we are interested in a configuration where the spins of particles 1 and 4 are the same in their rest frame as are those of particles 2 and 3. Because we need to compute at non-zero momentum transfer, the spins of particles 1 and 4 (and 2 and 3) will not be the same, rather they will differ by the fact that we need to boost their identical rest frame spins by different velocities. Thus with the previous center of mass kinematics the polarization vectors labelling the bosonic components of the left or right-moving states will be boosts of a polarization vector $\vec{\zeta}_1$ by $\vec{v}$ and $-\vec{w}$ respectively and similarly for particles 2 and 3. To the order in velocity required here this gives

$$\hat{p}_1 = M(1+v^2/2, \vec{v}), \quad \zeta_1 = (\vec{\zeta}_1 \cdot \vec{v}, \vec{\zeta}_1) \tag{13}$$

$$\hat{p}_2 = M(1+v^2/2, -\vec{v}), \quad \zeta_2 = (-\vec{\zeta}_2 \cdot \vec{v}, \vec{\zeta}_2) \tag{14}$$

$$\hat{p}_3 = -M(1+w^2/2, \vec{w}), \quad \zeta_3 = (\vec{\zeta}_2 \cdot \vec{w}, \vec{\zeta}_2) \tag{15}$$

$$\hat{p}_4 = -M(1+w^2/2, -\vec{w}), \quad \zeta_4 = (-\vec{\zeta}_1 \cdot \vec{w}, \vec{\zeta}_1) \tag{16}$$

Similar remarks apply to the spinors which label these states, that is we again take them to be equal for particles 1 and 4 and for 2 and 3 in the rest frame and then boost these spinors by $\pm v$, $\pm w$ to obtain the correct center of mass frame spinors. Our conventions for ten-dimensional gamma matrices are as follows. Define

$$\bar{\gamma}^i = i \begin{pmatrix} 0 & \gamma^i \\ \gamma^i & 0 \end{pmatrix} \tag{17}$$

where the γ^i are as in [8]. We take

$$\bar{\gamma}^9 = i \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \tag{18}$$

The 32 by 32 $SO(9,1)$ gamma matrices are then

$$\Gamma^0 = i \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}, \qquad \Gamma^m = \begin{pmatrix} 0 & \bar{\gamma}^m \\ \bar{\gamma}^m & 0 \end{pmatrix} \tag{19}$$

with $m = 1, \ldots 9$. Note that this representation is purely imaginary. We also define

$$\Gamma^{11} = -\Gamma^0 \Gamma^1 \cdots \Gamma^9 = diag(1, -1) \tag{20}$$

In IIB string theory we start with both left and right spinors tramsforming as the 16_+ of $SO(9,1)$, that is with Γ^{11} eigenvalue $+1$.

The Dirac equation is

$$\Gamma^\mu p_\mu^{L,R} u^{L,R}(p) = 0 \tag{21}$$

where it is to be understood that u transforms as 16_+ and hence can be written as the transpose of $(\lambda^{L,R}, 0)$ with $\lambda^{L,R}$ a 16 component spinor. In the rest frame we have $p^{R,L} = (M/2, \vec{0}, \pm R)$ so that

$$(M/2)(\Gamma^0 \mp \Gamma^9) u^{R,L} = 0 \tag{22}$$

which using the given basis for the gamma matrices is equivalent to the equation

$$-i\bar{\gamma}^9 \lambda^{R,L} = \mp \lambda^{R,L} \tag{23}$$

Thus if we define the 8_s of the $SO(8)$ little group of a massive particle to be states with $-i\bar{\gamma}^9$ eigenvalue $+1$ then this shows that $\lambda^R \sim 8_c$ and $\lambda^L \sim 8_s$. Note that we could also have used momentum states in the IIA string and would have found the same representations. Thus we conclude that the rest frame spinors are given by

$$\lambda^L = \sqrt{M/2} \begin{pmatrix} \xi_a^L \\ 0 \end{pmatrix}, \quad \lambda^R = \sqrt{M/2} \begin{pmatrix} 0 \\ \xi_{\dot{a}}^R \end{pmatrix} \tag{24}$$

Boosting then gives the left-handed spinors to order velocity squared:

$$\lambda_1^L = \sqrt{M/2} \begin{pmatrix} (1+v^2/4)\xi_1^L \\ -(\vec{v} \cdot \vec{\gamma}/2)\xi_1^L \end{pmatrix} \tag{25}$$

$$\lambda_2^L = \sqrt{M/2} \begin{pmatrix} (1+v^2/4)\xi_2^L \\ (\vec{v} \cdot \vec{\gamma}/2)\xi_2^L \end{pmatrix} \tag{26}$$

$$\lambda_3^L = \sqrt{M/2} \begin{pmatrix} (1+w^2/4)\xi_2^L \\ -(\vec{w} \cdot \vec{\gamma}/2)\xi_2^L \end{pmatrix} \tag{27}$$

$$\lambda_4^L = \sqrt{M/2} \begin{pmatrix} (1+w^2/4)\xi_1^L \\ (\vec{w} \cdot \vec{\gamma}/2)\xi_1^L \end{pmatrix} \tag{28}$$

with similar formulae for the right-moving components.

4. Four point string amplitude

The four point amplitude is calculated by sewing together two open string four point amplitudes using the formalism of [9]. This gives

$$A(1,2,3,4) = \kappa^2 \sin(\pi \hat{t}/8) A_4^L \times A_4^R \tag{29}$$

where $A_4^{L,R}$ are open string four point functions. In terms of nine-dimensional kinematics we have

$$A_4^L = \frac{\Gamma(-\hat{s}/8 + M^2/2)\Gamma(-\hat{t}/8)}{\Gamma(1-\hat{t}/8-\hat{s}/8+M^2/2)} K^L(1,2,3,4) \tag{30}$$

and

$$A_4^R = \frac{\Gamma(-\hat{t}/8)\Gamma(-\hat{u}/8)}{\Gamma(1-\hat{t}/8-\hat{u}/8)} K^R(1,2,3,4) \tag{31}$$

where $K^{L,R}(1,2,3,4)$ are kinematical factors depending on the momenta, polarization vectors, and spinors depending on whether the left or right-moving amplitude involves scattering of four vector states, four fermion states, or two vectors and two fermions. We will denote these as $K^L(\zeta_{1L}, \zeta_{2L}, \zeta_{3L}, \zeta_{4L})$, ${}^L K(u_{1L}, u_{2L}, u_{3L}, u_{4L})$ and $K^L(u_{1L}, \zeta_{2L}, \zeta_{3L}, u_{4L})$ respectively and similarly for the right-moving factor.

Expanding the sine and gamma functions in the non-relativistic limit gives

$$A(1,2,3,4) \sim \tag{32}$$
$$\frac{\kappa^2 \pi 8^3}{\hat{u}\hat{t}(\hat{s}-4M^2)} K^L(1,2,3,4) K^R(1,2,3,4) \tag{33}$$

In order to extract the long-range interactions we need to extract the $\hat{t}$ channel poles in this expression and then Fourier transform with respect to $\vec{q}$. As a result, any terms in $K^{L,R}$ which are proportional to $\hat{t}$ will not contribute to the $\hat{t}$ channel pole and can be dropped for the purpose of extracting the long-range interactions (they may contribute to contact interactions).

As an example consider the interaction of two states arising from the $(NS)^2$ sector. In this case we need the non-relativstic expansion of $K^L(\zeta_{1L}, \zeta_{2L}, \zeta_{3L}, \zeta_{4L}) K^R(\zeta_{1R}, \zeta_{2R}, \zeta_{3R}, \zeta_{4R})$. Using the previous expansions for the polarization tensors and momenta and dropping terms proportional to $\hat{t}$ we find

$$K^L(\zeta_{1L}, \zeta_{2L}, \zeta_{3L}, \zeta_{4L}) = \frac{\hat{u}}{64} \times \tag{34}$$
$$\left[\vec{k}^2 - 2(\vec{q}\cdot\vec{\zeta}_{1L})^2 - 2(\vec{q}\cdot\vec{\zeta}_{2L})^2\right] \tag{35}$$

For the scattering of two $(NS)^2$ states we then have for the t-channel pole

$$A_4 = -\frac{\pi\kappa^2}{8\vec{q}^2}\left[\vec{k}^2 - 2(\vec{q}\cdot\vec{\zeta}_1)^2 - 2(\vec{q}\cdot\vec{\zeta}_2)^2\right]^2 \tag{36}$$

where the square on the term in square brackets indicates the product of two terms with left and right-moving polarization vectors respectively.

Fourier transforming this amplitude with respect to $\vec{q}$, then gives the potential

$$V \sim \tag{37}$$
$$\left[4M^2\vec{v}^2 + 2(\vec{\nabla}\cdot\vec{\zeta}_1)^2 + 2(\vec{\nabla}\cdot\vec{\zeta}_2)^2\right]^2 \frac{1}{r^6} \tag{38}$$

This exhibits the spin-independent v^4 interaction, here going like $1/r^6$ instead of $1/r^7$ because we have dimensionally reduced to $D=9$. In addition we see that there is a spin-dependent term proportional to v^2 which goes like $1/r^8$ ($1/r^9$ in $D=10$) and a static spin-dependent term going like $1/r^{10}$ ($1/r^{11}$ in $D=10$). Note that since this expression is symmetric in ζ_L and ζ_R the spin-dependent terms vanish for $(NS)^2$ states in the **28** of $SO(8)$ since this occurs in the antisymmetric tensor product of $\mathbf{8_v} \times \mathbf{8_v}$.

As a second example consider the interaction of two bosons which arise from the $(R)^2$ sector. For these states we need the non-relativistic expansion of

$$K^L(u_{1L}, u_{2L}, u_{3L}, u_{4L}) K^R(u_{1R}, u_{2R}, u_{3R}, u_{4R}) \tag{39}$$

with

$$K(u_1, u_2, u_3, u_4) = \left[(-\hat{s}/8 + M^2/2)\bar{u}_2\Gamma^\mu u_3 \bar{u}_1 \Gamma_\mu u_4\right] \tag{40}$$

where we have again dropped terms proportional to $\hat{t}$.

Using the previous expressions for the spinor fields one finds after some algebra that

$$K^L(u_{1L}, u_{2L}, u_{3L}, u_{4L}) = \frac{\hat{s}-4M^2}{8} \times \tag{41}$$
$$\left[\frac{\vec{k}^2}{8} + \frac{5}{16}k_i q_j (R_{1L}^{ij} + R_{2L}^{ij}) - \frac{1}{16} q_j q_k R_{1L}^{ij} R_{2L}^{ik}\right] \tag{42}$$

and similarly for right-movers where $R_{1L}^{ij} = \xi_{1L}^\dagger \gamma^{ij} \xi_{1L}/4$. Combining left and right-moving

parts then gives for the four point amplitude

$$A_4 = -\frac{\pi\kappa^2}{8\vec{q}^2}\times \tag{43}$$

$$\left[\vec{k}^2 + \frac{5}{2}k_i q_j (R_1^{ij} + R_2^{ij}) - \frac{1}{2}q_j q_k R_1^{ij} R_2^{ik}\right]^2 \tag{44}$$

We again see the universal spin-independent v^4 term as in the expression for scattering of $(NS)^2$ states and additional spin dependent terms involving v^3, v^2, v, and a static term. Extrapolating to $D = 10$ the Fourier transform of A_4 has the schematic form

$$V \sim \frac{v^4}{r^7} + \frac{v^3 R}{r^8} + \frac{v^2 R^2}{r^9} + \frac{v R^3}{r^{10}} + \frac{R^4}{r^{11}} \tag{45}$$

with R a tensor bilinear in spinor fields.

Scattering of fermionic D0-branes in the R-NS sector can also be computed in an analogous manner. We consider scattering of states in the $NS - R$ sector with states in the $NS - R$ sector. The four point amplitude then factorizes as in eqn. (34) with K_L given by eqn. (35) and K_R by eqn. (41). The four point amplitude expanded in velocity as before is then

$$A_4 = -\frac{\pi\kappa^2}{8\vec{q}^2}\times \tag{46}$$

$$\left[\vec{k}^2 - 2(\vec{q}\cdot\vec{\zeta}_1)^2 - 2(\vec{q}\cdot\vec{\zeta}_2)^2\right]\times \tag{47}$$

$$\left[\vec{k}^2 + \frac{5}{2}k_i q_j (R_1^{ij} + R_2^{ij}) - \frac{1}{2}q_j q_k R_1^{ij} R_2^{ik}\right] \tag{48}$$

Fourier transforming with respect to $\vec{q}$ then gives the spin dependent potential between fermionic D0-branes.

5. Comparison with Matrix theory

These interactions should also be computable from the Matrix model proposed in [6] by generalizing the one-loop computation of [5,7] to include the fermion terms which are related to the v^4/r^7 term by supersymmetry. Such a computation has not yet appeared in the literature, so here we will be content to compare these results with the structure of the matrix model which can be determined from simple scaling arguments.

If we rescale fields so that the matrix model action takes the form (with ϕ the boson fields, ψ the fermion fields and ∂ denoting time derivative)

$$S = \frac{1}{g_s}\int dt\left((\partial\phi)^2 + \phi^4 + \psi\partial\psi + \psi^2\phi\right) \tag{49}$$

then we can do dimensional analysis by assigning the following dimensions to the fields, couplings, and derivatives:

$$[g_s] = -3, \quad [\phi] = -1, \quad [\partial] = -1, \quad [\psi] = -3/2 \tag{50}$$

The loop expansion for the effective action then has the form

$$S = {g_s}^{-1} S_0 + S_1 + g_s S_2 + \cdots \tag{51}$$

and the Lagrangian density $\mathcal{L}_i$ has dimension $-4 + 3i$.

We can also organize terms by the parameter $N = n_\partial + 2n_f$ with n_∂ the number of time derivatives and n_f the number of fermion fields. The supersymmetry transformations preserve N so terms paired by supersymmetry must all have the same value of N. The v^4/r^7 term occurs at one-loop and thus has dimension -1. It also cleary has $N = 4$. If all the terms we have computed have dimension -1 and $N = 4$ then it is plausible that they all arise from supersymmetrization of the v^4/r^7 term. To check this note that in the matrix model both the spinor bilinear R and any polarization tensors ζ have to arise as expectation values of operators which are bilinear in the fermion fields ψ. The potential terms we have computed when extrapolated to $D = 10$ then all have dimension -1 and $N = 4$ and match on to terms in the one-loop matrix model effective action of the schematic form

$$\mathcal{L}_1 \sim \frac{v^4}{r^7} + \frac{v^3\psi^2}{r^8} + \frac{v^2\psi^4}{r^9} + \frac{v\psi^6}{r^{10}} + \frac{\psi^8}{r^{11}} \tag{52}$$

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